MATHEMATICS

Cardinal Newman CATHOLIC SCHOOL

Year 11 into Year 12

WELCOME TO A Level MATHEMATICS AT CARDINAL NEWMAN CATHOLIC SCHOOL

A Level Maths is widely recognised as a highly valued A Level and will open many `doors' for you. Students who study Maths at this level are regarded as being the `elite'.

However, **A Level Maths is NOT an easy option** – it does require a lot of self- motivation, determination and self-study. We recommend that you do a minimum of 6 hours' work outside the classroom each week. You will need to `love a challenge' and be willing to accept that a question has `gone wrong' – and be prepared to have another attempt (and another and maybe even another). A Level Maths is a two-year course.

Year 1 Mathematics	
Paper 1:Pure Mathematics (internal examination in June) Written examination: 2 hours 66.66% of the qualification 100 marks	**Content overview**: Proof, Algebra and functions, Coordinate Geometry in the (x, y) plane, Sequences and Series, Trigonometry, Exponentials and logarithms, Differentiation, Integration, Vectors
Paper 2: Statistics & Mechanics (internal examination in June) Written examination: 1 hour 33.33% of the qualification 50 marks	**Content overview** **Section A: Statistics** ● Topic 1 – Statistical sampling ● Topic 2 – Data presentation and interpretation ● Topic 3 – Probability ● Topic 4 – Statistical distributions ● Topic 5 – Statistical hypothesis testing **Section B: Mechanics** ● Topic 6 – Quantities and units in mechanics ● Topic 7 – Kinematics ● Topic 8 – Forces and Newton's laws

Year 2 Mathematics	
Paper 1: Pure Mathematics 1 Written examination: 2 hours 33.33% of the qualification 100 marks	AS level pure mathematics content – the same content as AS Paper 1 but tested at A level demand
Paper 2: Pure Mathematics 2 Written examination: 2 hours 33.33% of the qualification 100 marks	**Content overview** ● Topic 1 – Proof ● Topic 2 – Algebra and functions ● Topic 3 – Coordinate geometry in the (x,y) plane ● Topic 4 – Sequences and series ● Topic 5 – Trigonometry ● Topic 6 – Differentiation ● Topic 7 – Integration ● Topic 8 – Numerical methods
Paper 3: Statistics and Mechanics Written examination: 2 hours 33.33% of the qualification 100 marks	**Content overview** **Section A: Statistics** ● Topic 1 – Statistical sampling ● Topic 2 – Data presentation and interpretation ● Topic 3 – Probability ● Topic 4 – Statistical distributions ● Topic 5 – Statistical hypothesis testing **Section B: Mechanics** ● Topic 6 – Quantities and units in mechanics ● Topic 7 – Kinematics ● Topic 8 – Forces and Newton's laws ● Topic 9 – Moments

Any student who wishes to follow this course should have followed the Higher Tier GCSE in set 1 and obtained at least a grade 7.

Those with a grade 6 (from set 1) will have to sit an Entrance Test in September to evaluate their Algebra skills mainly.

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Thank you for choosing to study Mathematics in the sixth form. The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. In order that you make the best possible start to the course, we have prepared this booklet.

It is vitally important that you spend some time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics before you commence your course in September.

You should have met all the topics before at GCSE. Work through the introduction to each chapter, making sure that you understand the examples. Then tackle the exercise – not necessarily every question, but enough to ensure you understand the topic thoroughly.

We will test you at the start of September where the answers will be given out, to check how well you understand these topics. They are the first two chapters of the A-level Year 1 programme; so it is important that you have looked at all the booklet before then. If you do not pass this test, you will be provided with a programme of additional work in order to bring your basic algebra skills to the required standard. A mock test is provided at the back of this booklet.

We hope that you will use this introduction to give you a good start to your AS work and that it will help you enjoy and benefit from the course more.

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WORK TO BE COMPLETED BY SEPTEMBER :

Rules of indices: Page 6 to 7,

Using Surds: Page 10 to 11,

Solving quadratics by completing the square: Page 16 to 18,

Using the Discriminant of a quadratic equation: Page 21,

Solving quadratic inequalities and simultaneous equations: Page 32 to 33,

Coordinates Geometry (Straight line): Page 44 to 49

Extension: Page 50 to 52

1.2 You can simplify expressions and functions by using rules of indices (powers).

- $a^m \times a^n = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $a^{-m} = \dfrac{1}{a^m}$
  $a^{\frac{1}{m}} = \sqrt[m]{a}$ — The mth root of a.
  $a^{\frac{n}{m}} = \sqrt[m]{a^n}$

Example 2

Simplify these expressions:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $b^4 \div b^4$
d $6x^{-3} \div 3x^{-5}$ **e** $(a^3)^2 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

a $x^2 \times x^5$
$= x^{2+5}$ — Use the rule $a^m \times a^n = a^{m+n}$ to simplify the index.
$= x^7$

b $2r^2 \times 3r^3$
$= 2 \times 3 \times r^2 \times r^3$ — Rewrite the expression with the numbers together and the r terms together.
$= 6 \times r^{2+3}$ — $2 \times 3 = 6$, $r^2 \times r^3 = r^{2+3}$
$= 6r^5$

c $b^4 \div b^4$ — Use the rule $a^m \div a^n = a^{m-n}$
$= b^{4-4}$
$= b^0 = 1$ — Any term raised to the power of zero = 1.

d $6x^{-3} \div 3x^{-5}$
$= 6 \div 3 \times x^{-3} \div x^{-5}$ — $x^{-3} \div x^{-5} = x^{-3--5} = x^2$
$= 2 \times x^2$
$= 2x^2$

e $(a^3)^2 \times 2a^2$ — Use the rule $(a^m)^n = a^{mn}$ to simplify the index.
$= a^6 \times 2a^2$ — $a^6 \times 2a^2 = 1 \times 2 \times a^6 \times a^2 = 2 \times a^{6+2}$
$= 2 \times a^6 \times a^2$
$= 2 \times a^{6+2}$
$= 2a^8$

f $(3x^2)^3 \div x^4$ — Use the rule $(a^m)^n = a^{mn}$ to simplify the index.
$= 27x^6 \div x^4$
$= 27 \div 1 \times x^6 \div x^4$
$= 27 \times x^{6-4}$
$= 27x^2$

1.6 You can extend the rules of indices to all rational exponents.

- $a^m \times a^n = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $a^{\frac{1}{m}} = \sqrt[m]{a}$
  $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
  $a^{-m} = \dfrac{1}{a^m}$
  $a^0 = 1$

Hint: Rational numbers can be written as $\frac{a}{b}$ where a and b are both integers, e.g. -3.5, $1\frac{1}{4}$, 0.9, 7, $0.1\dot{3}$

Example 6

Simplify:

a $x^4 \div x^{-3}$

b $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$

c $(x^3)^{\frac{2}{3}}$

d $2x^{1.5} \div 4x^{-0.25}$

a $x^4 \div x^{-3}$
$= x^{4--3}$ — Use the rule $a^m \div a^n = a^{m-n}$. Remember $- + - = +$.
$= x^7$

b $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ — This could also be written as $\sqrt{x}$.
$= x^{\frac{1}{2}+\frac{3}{2}}$ — Use the rule $a^m \times a^n = a^{m+n}$.
$= x^2$

c $(x^3)^{\frac{2}{3}}$ — Use the rule $(a^m)^n = a^{mn}$.
$= x^{3\times\frac{2}{3}}$
$= x^2$

d $2x^{1.5} \div 4x^{-0.25}$
$= \frac{1}{2}x^{1.5--0.25}$ — Use the rule $a^m \div a^n = a^{m-n}$. $2 \div 4 = \frac{1}{2}$. $1.5 - -0.25 = 1.75$
$= \frac{1}{2}x^{1.75}$

Example 7

Evaluate:

a $9^{\frac{1}{2}}$

b $64^{\frac{1}{3}}$

c $49^{\frac{3}{2}}$

d $25^{-\frac{3}{2}}$

a $9^{\frac{1}{2}}$

$= \sqrt{9}$ — Using $a^{\frac{1}{m}} = \sqrt[m]{a}$.

$= \pm 3$ — When you take a square root, the answer can be positive or negative as $+ \times + = +$ and $- \times - = +$.

b $64^{\frac{1}{3}}$

$= \sqrt[3]{64}$ — This means the cube root of 64.

$= 4$ — As $4 \times 4 \times 4 = 64$.

c $49^{\frac{3}{2}}$

$= (\sqrt{49})^3$ — Using $a^{\frac{n}{m}} = \sqrt[m]{a^n}$. This means the square root of 49, cubed.

$= \pm 343$

d $25^{-\frac{3}{2}}$

$= \dfrac{1}{25^{\frac{3}{2}}}$ — Using $a^{-m} = \dfrac{1}{a^m}$.

$= \dfrac{1}{(\pm\sqrt{25})^3}$ — $\sqrt{25} = \pm 5$

$= \dfrac{1}{(\pm 5)^3}$

$= \pm \dfrac{1}{125}$

TO DO:

1. Given that

$$y = \frac{1}{27}x^3$$

express each of the following in the form $k\,x^n$ where k and n are constants.

(*a*) $y^{\frac{1}{3}}$

(1)

(*b*) $3y^{-1}$

(1)

(*c*) $\sqrt{(27y)}$

(1)

---

2. Simplify the following expressions fully.

(*a*) $\left(\frac{1}{9}x^4\right)^{0.5}$

(1)

(*b*) $\left(\frac{x}{\sqrt{2}}\right)^{-2}$

(1)

(*c*) $x\sqrt{3}$, $\sqrt{\frac{48}{x^4}}$

(2)

---

3. Given that $32\sqrt{2} = 2^a$, find the value of a.

(3)

---

4. (*a*) Find the value of $16^{-\frac{1}{4}}$.

(2)

(*b*) Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$.

(2)

5.

Solve the equation

$$9^x = 3^{x+2}.$$ [3]

---

6. (*a*) Evaluate $(32)^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(*b*) Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$.

(2)

7.

Solve the equations

(i) $3^m = 81,$ [1]

(ii) $(36p^4)^{\frac{1}{2}} = 24,$ [3]

(iii) $5^n \times 5^{n+4} = 25.$ [3]

8.

Express each of the following in the form 3^n:

(i) $\frac{1}{9}$, [1] (ii) $\sqrt[3]{3}$, [1] (iii) $3^{10} \times 9^{15}$.[2]

9.

Solve the equations

(i) $10^p = 0.1,$ [1]

(ii) $(25k^2)^{\frac{1}{2}} = 15,$ [3]

(iii) $t^{-\frac{1}{3}} = \frac{1}{2}.$ [2]

10.

Solve the equations

(i) $x^{\frac{1}{3}} = 2,$ [1]

(ii) $10^t = 1,$ [1]

(iii) $(y^{-2})^2 = \frac{1}{81}.$ [2]

---

11. Solve

(*a*) $2^y = 8,$

(1)

(*b*) $2^x \times 4^{x+1} = 8.$ **(4)**

1.7 You can write a number exactly using surds, e.g. $\sqrt{2}$, $\sqrt{3} - 5$, $\sqrt{19}$.
You cannot evaluate surds exactly because they give never-ending, non-repeating decimal fractions, e.g. $\sqrt{2} = 1.414\,213\,562\ldots$
The square root of a prime number is a surd.

■ You can manipulate surds using these rules:

$$\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Example 8

Simplify:

a $\sqrt{12}$ **b** $\frac{\sqrt{20}}{2}$ **c** $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

a $\sqrt{12}$

$= \sqrt{(4 \times 3)}$

$= \sqrt{4} \times \sqrt{3}$ — Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$. $\sqrt{4} = 2$

$= 2\sqrt{3}$

b $\frac{\sqrt{20}}{2}$ — $\sqrt{20} = \sqrt{4} \times \sqrt{5}$

$= \frac{\sqrt{4} \times \sqrt{5}}{2}$ — $\sqrt{4} = 2$

$= \frac{2 \times \sqrt{5}}{2}$ — Cancel by 2.

$= \sqrt{5}$

c $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

$= 5\sqrt{6} - 2\sqrt{6}\sqrt{4} + \sqrt{6} \times \sqrt{49}$ — $\sqrt{6}$ is a common factor.

$= \sqrt{6}(5 - 2\sqrt{4} + \sqrt{49})$ — Work out the square roots $\sqrt{4}$ and $\sqrt{49}$.

$= \sqrt{6}(5 - 2 \times 2 + 7)$ — $5 - 4 + 7 = 8$

$= \sqrt{6}(8)$

$= 8\sqrt{6}$

1.8 You rationalise the denominator of a fraction when it is a surd.

■ The rules to rationalise surds are:

- Fractions in the form $\sqrt{\frac{1}{a}}$, multiply the top and bottom by $\sqrt{a}$.
- Fractions in the form $\frac{1}{a+\sqrt{b}}$, multiply the top and bottom by $a-\sqrt{b}$.
- Fractions in the form $\frac{1}{a-\sqrt{b}}$, multiply the top and bottom by $a+\sqrt{b}$.

Example 9

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{3+\sqrt{2}}$ **c** $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

a $\frac{1}{\sqrt{3}}$

$= \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

Multiply the top and bottom by $\sqrt{3}$.
$\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$

b $\frac{1}{3+\sqrt{2}}$

$= \frac{1 \times (3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$

$= \frac{3-\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-2}$

$= \frac{3-\sqrt{2}}{7}$

Multiply top and bottom by $(3-\sqrt{2})$.
$\sqrt{2} \times \sqrt{2} = 2$
$9-2=7$, $-3\sqrt{2}+3\sqrt{2}=0$

c $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$

$= \frac{5+\sqrt{5}\sqrt{2}+\sqrt{2}\sqrt{5}+2}{5-2}$

$= \frac{7+2\sqrt{10}}{3}$

Multiply top and bottom by $\sqrt{5}+\sqrt{2}$.
$-\sqrt{2}\sqrt{5}$ and $\sqrt{5}\sqrt{2}$ cancel each other out.
$\sqrt{5}\sqrt{2} = \sqrt{10}$

TO DO:

1. (*a*) Write $\sqrt{45}$ in the form $a\sqrt{5}$, where a is an integer.

(1)

(*b*) Express $\dfrac{2(3+\sqrt{5})}{(3-\sqrt{5})}$ in the form $b + c\sqrt{5}$, where b and c are integers.

(5)

---

2. (*a*) Expand and simplify $(4 + \sqrt{3})(4 - \sqrt{3})$.

(2)

(*b*) Express $\dfrac{26}{4+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

(2)

---

3. Answer this question without a calculator, showing all your working and giving your answers in their simplest form.

(i) Solve the equation

$$4^{2x+1} = 8^{4x}$$

(3)

(ii) (a) Express

$$3\sqrt{18} - \sqrt{32}$$

in the form $k\sqrt{2}$, where k is an integer.

(2)

(b) Hence, or otherwise, solve

$$3\sqrt{18} - \sqrt{32} = \sqrt{n}$$

(2)

---

4. (*a*) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(1)

(*b*) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found.

(3)

---

5. Simplify

$$\frac{5-\sqrt{3}}{2+\sqrt{3}},$$

Giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(4)

---

6. Simplify $(3 + \sqrt{5})(3 - \sqrt{5})$. **(2)**

---

7. Simplify

(*a*) $(3\sqrt{7})^2$

(1)

(*b*) $(8 + \sqrt{5})(2 - \sqrt{5})$

(3)

---

8. Simplify

$$\frac{5 - 2\sqrt{3}}{\sqrt{3} - 1},$$

giving your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers.

(4)

9.

Express each of the following in the form $k\sqrt{2}$, where k is an integer:

(i) $\sqrt{200}$, [1]

(ii) $\dfrac{12}{\sqrt{2}}$, [1]

(iii) $5\sqrt{8} - 3\sqrt{2}$. [2]

10.

Simplify the following, expressing each answer in the form $a\sqrt{5}$.

(i) $3\sqrt{10} \times \sqrt{2}$ [2]

(ii) $\sqrt{500} + \sqrt{125}$ [3]

11.

(i) Evaluate $27^{-\frac{2}{3}}$. [2]

(ii) Express $5\sqrt{5}$ in the form 5^n. [1]

(iii) Express $\dfrac{1 - \sqrt{5}}{3 + \sqrt{5}}$ in the form $a + b\sqrt{5}$. [3]

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12. Without using your calculator, solve

$$x\sqrt{27} + 21 = \frac{6x}{\sqrt{3}}$$

Write your answer in the form $a\sqrt{b}$ where a and b are integers.

You must show all stages of your working. **(4)**

2.1 You need to be able to plot graphs of quadratic equations.

■ **The general form of a quadratic equation is**

$$y = ax^2 + bx + c$$

where a, b and c are constants and $a \neq 0$.

This could also be written as $f(x) = ax^2 + bx + c$.

Example 1

a Draw the graph with equation $y = x^2 - 3x - 4$ for values of x from -2 to $+5$.

b Write down the minimum value of y and the value of x for this point.

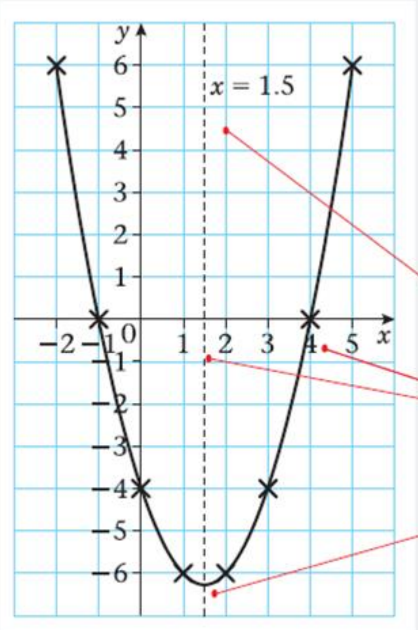
c Label the line of symmetry.

a

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$-3x$	$+6$	$+3$	0	-3	-6	-9	-12	-15
-4	-4	-4	-4	-4	-4	-4	-4	-4
y	6	0	-4	-6	-6	-4	0	6

① First draw a table of values. Remember any number squared is positive.

② Look at the table to determine the extent of the y-axis. Use values of y from -6 to $+6$.



③ Plot the points and then join all the points together with a smooth curve.

The general shape of the curve is a ∪, it is called a parabola.

This is the line of symmetry. It is always half-way between the x-axis crossing points. It has equation $x = 1.5$.

This is the minimum.

b Minimum value is $y = -6.3$ when $x = 1.5$.

c See graph.

2.2 You can solve quadratic equations using factorisation.

Quadratic equations have two solutions or roots. (In some cases the two roots are equal.)
To solve a quadratic equation, put it in the form $ax^2 + bx + c = 0$.

Example 2

Solve the equation $x^2 = 9x$

$x^2 = 9x$

$x^2 - 9x = 0$ — Rearrange in the form $ax^2 + bx + c = 0$.

$x(x - 9) = 0$ — Factorise by x (factorising is in Chapter 1). Then either part of the product could be zero.

Then either $x = 0$

or $x - 9 = 0 \Rightarrow x = 9$

So $x = 0$ or $x = 9$ are the two solutions of the equation $x^2 = 9x$.

A quadratic equation has two solutions (roots). In some cases the two roots are equal.

Example 3

Solve the equation $x^2 - 2x - 15 = 0$

$x^2 - 2x - 15 = 0$

$(x + 3)(x - 5) = 0$ — Factorise.

Then either $x + 3 = 0 \Rightarrow x = -3$

or $x - 5 = 0 \Rightarrow x = 5$

The solutions are $x = -3$ or $x = 5$.

Example 4

Solve the equation $6x^2 + 13x - 5 = 0$

$6x^2 + 13x - 5 = 0$

$(3x - 1)(2x + 5) = 0$ — Factorise.

Then either $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

or $2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$

The solutions are $x = \frac{1}{3}$ or $x = -\frac{5}{2}$.

The solutions can be fractions or any other type of number.

Example 5

Solve the equation $x^2 - 5x + 18 = 2 + 3x$

$x^2 - 5x + 18 = 2 + 3x$

$x^2 - 8x + 16 = 0$ — Rearrange in the form $ax^2 + bx + c = 0$.

$(x - 4)(x - 4) = 0$ — Factorise.

Then either $x - 4 = 0 \Rightarrow x = 4$

or $x - 4 = 0 \Rightarrow x = 4$

$\Rightarrow x = 4$

Here $x = 4$ is the only solution, i.e. the two roots are equal.

Example 6

Solve the equation $(2x - 3)^2 = 25$

$(2x - 3)^2 = 25$
$2x - 3 = \pm 5$
$2x = 3 \pm 5$
Then either $2x = 3 + 5 \Rightarrow x = 4$
or $2x = 3 - 5 \Rightarrow x = -1$
The solutions are $x = 4$ or $x = -1$.

This is a special case.
Take the square root of both sides.
Remember $\sqrt{25} = +5$ or -5.
Add 3 to both sides.

Example 7

Solve the equation $(x - 3)^2 = 7$

$(x - 3)^2 = 7$
$x - 3 = \pm\sqrt{7}$
$x = +3 \pm \sqrt{7}$
Then either $x = 3 + \sqrt{7}$
or $x = 3 - \sqrt{7}$
The solutions are $x = 3 + \sqrt{7}$ or $x = 3 - \sqrt{7}$.

Square root. (If you do not have a calculator, leave this in surd form.)

2.3 You can write quadratic expressions in another form by completing the square.

In general

■ **Completing the square:** $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

Example 9

Complete the square for the expressions

a $x^2 + 12x$ **b** $2x^2 - 10x$

a $x^2 + 12x$
$= (x + 6)^2 - 6^2$
$= (x + 6)^2 - 36$

$2b = 12$, so $b = 6$

b $2x^2 - 10x$
$= 2(x^2 - 5x)$
$= 2[(x - \frac{5}{2})^2 - (\frac{5}{2})^2]$
$= 2(x - \frac{5}{2})^2 - \frac{25}{2}$

Here the coefficient of x^2 is 2.
So take out the coefficient of x^2.
Complete the square on $(x^2 - 5x)$.
Use $b = -5$.

2.4 You can solve quadratic equations by completing the square.

Example 10

Solve the equation $x^2 + 8x + 10 = 0$ by completing the square.

$x^2 + 8x + 10 = 0$	Check coefficient of $x^2 = 1$.
$x^2 + 8x = -10$	Subtract 10 to get LHS in the form $ax^2 + b$.
$(x + 4)^2 - 4^2 = -10$	Complete the square for $(x^2 + 8x)$.
$(x + 4)^2 = -10 + 16$	Add 4^2 to both sides.
$(x + 4)^2 = 6$	
$(x + 4) = \pm\sqrt{6}$	Square root both sides.
$x = -4 \pm \sqrt{6}$	Subtract 4 from both sides.
Then the solutions (roots) of $x^2 + 8x + 10 = 0$ are either $x = -4 + \sqrt{6}$ or $x = -4 - \sqrt{6}$.	Leave your answer in surd form as this is a non-calculator question.

Example 11

Solve the equation $2x^2 - 8x + 7 = 0$.

$2x^2 - 8x + 7 = 0$	The coefficient of $x^2 = 2$.
$x^2 - 4x + \frac{7}{2} = 0$	So divide by 2.
$x^2 - 4x = -\frac{7}{2}$	Subtract $\frac{7}{2}$ from both sides.
$(x - 2)^2 - (2)^2 = -\frac{7}{2}$	Complete the square for $x^2 - 4x$.
$(x - 2)^2 = -\frac{7}{2} + 4$	Add $(2)^2$ to both sides.
$(x - 2)^2 = \frac{1}{2}$	Combine the RHS.
$x - 2 = \pm\sqrt{\frac{1}{2}}$	Square root both sides.
$x = 2 \pm \frac{1}{\sqrt{2}}$	Add 2 to both sides.

So the roots are either

$$x = 2 + \frac{1}{\sqrt{2}}$$

or $x = 2 - \frac{1}{\sqrt{2}}$

TO DO:

1. $x^2 - 8x - 29 \equiv (x + a)^2 + b$,

where a and b are constants.

(a) Find the value of a and the value of b.

(3)

(b) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$, where c and d are integers to be found.

(3)

2.

(i) Express $x^2 + 8x + 18$ in the form $(x + a)^2 + b$. [2]

(ii) Sketch the graph of $y = x^2 + 8x + 18$, stating the coordinates of its vertex. [3]

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3 (a) (i) Express $x^2 - 4x + 9$ in the form $(x - p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. *(2 marks)*

4.

(a) (i) Express $x^2 + 10x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Write down the coordinates of the vertex (minimum point) of the curve with equation $y = x^2 + 10x + 19$. *(2 marks)*

(iii) Write down the equation of the line of symmetry of the curve $y = x^2 + 10x + 19$. *(1 mark)*

(iv) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 10x + 19$. *(3 marks)*

(b) Determine the coordinates of the points of intersection of the line $y = x + 11$ and the curve $y = x^2 + 10x + 19$. *(4 marks)*

5.

Express $2x^2 + 12x + 13$ in the form $a(x + b)^2 + c$. [4]

6.

(i) Find the constants a, b and c such that, for all values of x,

$$4x^2 + 40x + 97 = a(x+b)^2 + c.$$ [4]

(ii) Hence write down the equation of the line of symmetry of the curve $y = 4x^2 + 40x + 97$. [1]

7.

(i) Find the constants a and b such that, for all values of x,

$$x^2 + 6x + 20 = (x+a)^2 + b.$$ [3]

(ii) Hence state the least value of $x^2 + 6x + 20$, and state also the value of x for which this least value occurs. [2]

(iii) Write down the greatest value of $\frac{1}{x^2 + 6x + 20}$. [1]

8.

(i) Express $2x^2 + 4x - 1$ in the form

$$a[(x+p)^2 + q],$$

stating the values of the constants a, p and q. [4]

(ii) Sketch the graph of $y = 2x^2 + 4x - 1$, stating the coordinates of the vertex. [4]

(iii) The graph of $y = 2x^2 + 4x - 1$ is obtained from the graph of $y = x^2$ by a sequence of transformations. Describe such a sequence, specifying each transformation fully, and stating the order in which they are applied. [4]

---

9. $4x - 5 - x^2 = q - (x+p)^2$,

where p and q are integers.

(a) Find the value of p and the value of q.

(3)

(b) Calculate the discriminant of $4x - 5 - x^2$.

(2)

(c) Sketch the curve with equation $y = 4x - 5 - x^2$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

10.

(a) Express $5x^2 - 20x + 3$ in the form $p(x+q)^2 + r$, where p, q and r are integers. **[3]**

(b) State the coordinates of the minimum point of the curve $y = 5x^2 - 20x + 3$. **[2]**

(c) State the equation of the normal to the curve $y = 5x^2 - 20x + 3$ at its minimum point. **[1]**

---

11. Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(*a*) express f(x) in the form $(x - a)^2 + b$, where a and b are integers.

(3)

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(*b*) Sketch the graph of C, showing the coordinates of P and Q. **(4)**

The line $y = 41$ meets C at the point R.

(*c*) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

(5)

2.6 You need to be able to sketch graphs of quadratic equations and solve problems using the discriminant.

The steps to help you sketch the graphs are:

1 Decide on the shape.

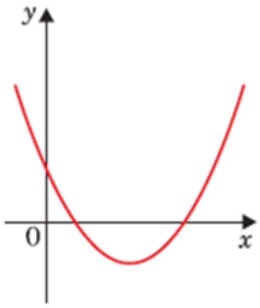
When a is >0 the curve will be a  shape.

When a is <0 the curve will be a  shape.

2 Work out the points where the curve crosses the x- and y-axes.
Put $y = 0$ to find the x-axis crossing points coordinates.
Put $x = 0$ to find the y-axis crossing points coordinates.

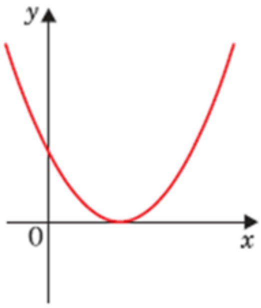
3 Check the general shape of curve by considering the discriminant, $b^2 - 4ac$.
When specific conditions apply, the general shape of the curve takes these forms:

$b^2 > 4ac$ and $a > 0$



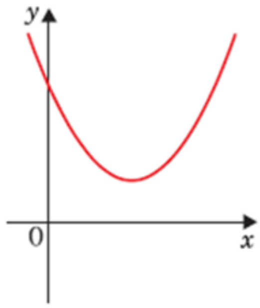
Here there are two different roots.

$b^2 = 4ac$ and $a > 0$



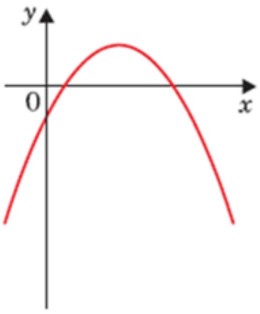
Here there are two equal roots.

$b^2 < 4ac$ and $a > 0$



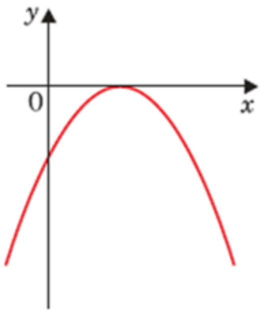
Here there are no real roots.

$b^2 > 4ac$ and $a < 0$



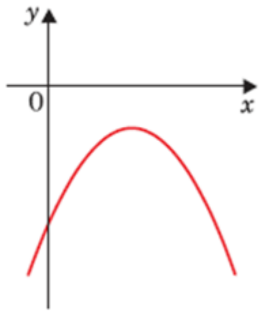
Here there are two different roots.

$b^2 = 4ac$ and $a < 0$



Here there are two equal roots.

$b^2 < 4ac$ and $a < 0$



Here there are no real roots.

You can use the discriminant to establish when a quadratic equation has

- equal roots: $b^2 = 4ac$
- real roots: $b^2 > 4ac$
- no real roots: $b^2 < 4ac$

Example 14

Sketch the graph of $y = x^2 - 5x + 4$

$a > 0$ so it is a ⋃ shape.

When $y = 0$,

$0 = x^2 - 5x + 4$

$0 = (x - 4)(x - 1)$

$x = 4$ or $x = 1$

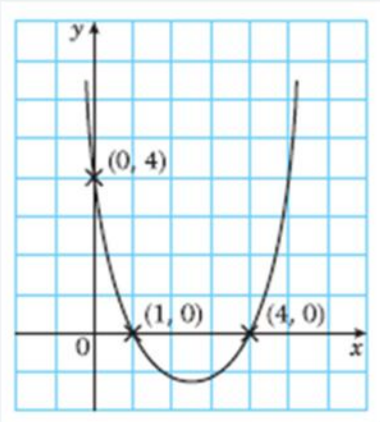
So x-axis crossing points are $(4, 0)$ and $(1, 0)$.

When $x = 0$, $y = 4$, so y-axis crossing point $= (0, 4)$

$b^2 = 25$, $4ac = 16$

So $b^2 > 4ac$ and $a > 0$.

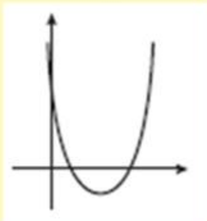
So sketch of the graph is:



Factorise to solve the equation. (You may need to use the formula or complete the square.)

$a = 1$, $b = -5$, $c = 4$

Remember general shape:



Label the crossing points.

Example 15

Find the values of k for which $x^2 + kx + 9 = 0$ has equal roots.

$x^2 + kx + 9 = 0$

Here $a = 1$, $b = k$ and $c = 9$

$k^2 = 4 \times 1 \times 9$

So $k = \pm 6$

For equal roots use $b^2 = 4ac$

Find the range of values of k for which $x^2 + 4x + k = 0$ has two distinct real solutions.

$x^2 + 4x + k = 0$

Here $a = 1$, $b = 4$ and $c = k$.

For two real solutions, $b^2 - 4ac > 0$

$4^2 - 4 \times 1 \times k > 0$

$16 - 4k > 0$

$16 > 4k$

$4 > k$

So $k < 4$

This statement involves an inequality, so your answer will also be an inequality.

For any value of k less than 4, the equation will have 2 distinct real solutions.

Online Explore how the value of the discriminant changes with k using GeoGebra.

TO DO:

1. The equation $2x^2 - 3x - (k + 1) = 0$, where k is a constant, has no real roots.

Find the set of possible values of k.

(4)

---

2. The equation $x^2 + kx + (k + 3) = 0$, where k is a constant, has different real roots.

(*a*) Show that $k^2 - 4k - 12 > 0$.

(2)

(*b*) Find the set of possible values of k. **(4)**

---

3. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(*a*) show that $q^2 + 8q < 0$.

(2)

(*b*) Hence find the set of possible values of q.

(3)

---

4 (a) Show that the equation

$$(2\sqrt{2} - 2)x^2 + \sqrt{8}x + (1 + \sqrt{2}) = 0$$

has two equal roots. **(3)**

(b) Hence, or otherwise, solve the equation

$$(2\sqrt{2} - 2)x^2 + \sqrt{8}x + (1 + \sqrt{2}) = 0$$

Give your answer in the form $a + b\sqrt{2}$, where a and b are rational numbers to be found. Show all of your working. **(3)**

---

5. The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(*a*) Show that k satisfies

$$k^2 + 2k - 3 > 0.$$

(3)

(*b*) Find the set of possible values of k. **(4)**

---

6. The straight line l has equation $y = k(2x - 1)$, where k is a constant.

The curve C has equation $y = x^2 + 2x + 11$

Find the set of values of k for which l does not cross or touch C. **(6)**

3.1 You can solve simultaneous linear equations by elimination.

Example 1

Solve the equations:

a $2x + 3y = 8$
$3x - y = 23$

b $4x - 5y = 4$
$6x + 2y = 25$

a $2x + 3y = 8$

$9x - 3y = 69$

$11x = 77$

$x = 7$

$14 + 3y = 8$

$3y = 8 - 14$

$y = -2$

So solution is $x = 7, y = -2$

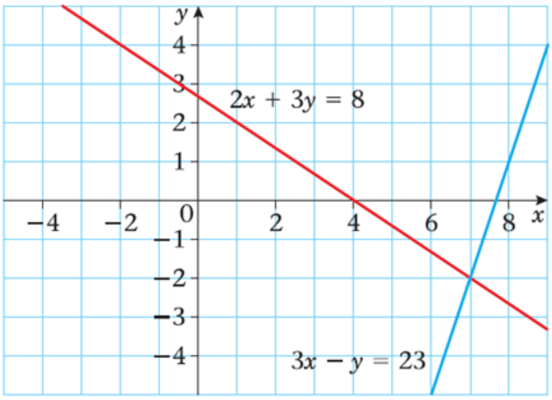
First look for a way to eliminate x or y.

Multiply the 2nd equation by 3 to get $3y$ in each equation.

Then add, since the $3y$ terms have different signs and y will be eliminated.

Use $x = 7$ in the first equation to find y.

You can consider the solution graphically.
The graph of each equation is a straight line.
The two straight lines intersect at $(7, -2)$.



b $12x - 15y = 12$

$12x + 4y = 50$

$-19y = -38$

$y = 2$

$4x - 10 = 4$

$4x = 14$

$x = 3\frac{1}{2}$

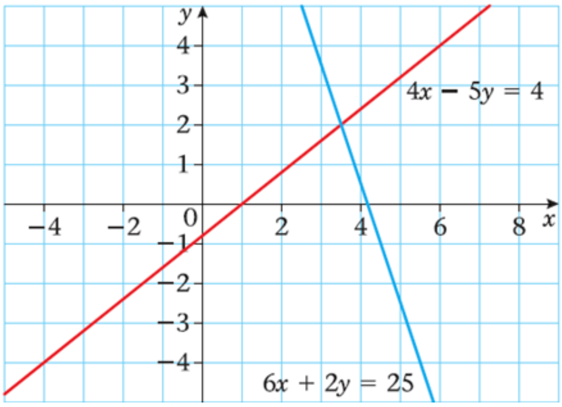
So solution is $x = 3\frac{1}{2}, y = 2$

Multiply the first equation by 3 and multiply the 2nd equation by 2 to get $12x$ in each equation.

Subtract, since the $12x$ terms have the same sign (both positive).

Use $y = 2$ in the first equation to find the value of x.

Graphically, each equation is a straight line.
The two straight lines intersect at (3.5, 2).



3.2 You can solve simultaneous linear equations by substitution.

Example 2

Solve the equations:

$$2x - y = 1$$
$$4x + 2y = -30$$

$y = 2x - 1$	Rearrange an equation to get either $x = \ldots$ or $y = \ldots$ (here $y = \ldots$).
$4x + 2(2x - 1) = -30$	Substitute this into the other equation (here in place of y).
$4x + 4x - 2 = -30$	Solve for x.
$8x = -28$	
$x = -3\frac{1}{2}$	
$y = 2(-3\frac{1}{2}) - 1 = -8$	Substitute $x = -3\frac{1}{2}$ into $y = 2x - 1$ to find the value of y.
So solution is $x = -3\frac{1}{2}$, $y = -8$.	

3.3 **You can use the substitution method to solve simultaneous equations where one equation is linear and the other is quadratic.**

Example 3

Solve the equations:

a $x + 2y = 3$
$x^2 + 3xy = 10$

b $3x - 2y = 1$
$x^2 + y^2 = 25$

a $x = 3 - 2y$

Rearrange the linear equation to get $x = \dots$ or $y = \dots$ (here $x = \dots$).

$(3 - 2y)^2 + 3y(3 - 2y) = 10$

Substitute this into the quadratic equation (here in place of x). $(3 - 2y)^2$ means $(3 - 2y)(3 - 2y)$ (see Chapter 1).

$9 - 12y + 4y^2 + 9y - 6y^2 = 10$

$-2y^2 - 3y - 1 = 0$

$2y^2 + 3y + 1 = 0$

$(2y + 1)(y + 1) = 0$

Solve for y using factorisation.

$y = -\frac{1}{2}$ or $y = -1$

So $x = 4$ or $x = 5$

Find the corresponding x-values by substituting the y-values into $x = 3 - 2y$.

Solutions are $x = 4$, $y = -\frac{1}{2}$

and $x = 5$, $y = -1$

There are two solution pairs. The graph of the linear equation (straight line) would intersect the graph of the quadratic (curve) at two points.

b $3x - 2y = 1$

$2y = 3x - 1$

$y = \frac{3x - 1}{2}$

Find $y = \dots$ from linear equation.

$x^2 + \left(\frac{3x - 1}{2}\right)^2 = 25$

Substitute $y = \frac{3x - 1}{2}$ into the quadratic equation to form an equation in x.

$x^2 + \left(\frac{9x^2 - 6x + 1}{4}\right) = 25$

Now multiply by 4.

$4x^2 + 9x^2 - 6x + 1 = 100$

$13x^2 - 6x - 99 = 0$

$(13x + 33)(x - 3) = 0$

$x = -\frac{33}{13}$ or $x = 3$

Solve for x.

$y = -\frac{56}{13}$ or $y = 4$

Substitute x-values into $y = \frac{3x - 1}{2}$.

Solutions are $x = 3$, $y = 4$

and $x = -\frac{33}{13}$, $y = -\frac{56}{13}$

3.4 You can solve linear inequalities using similar methods to those for solving linear equations.

■ **When you multiply or divide an inequality by a negative number, you need to change the inequality sign to its opposite.**

You need to be careful when you multiply or divide an inequality by a negative number. You need to turn round the inequality sign:

$$5 > 2$$

Multiply by -2 $\quad -10 < -4$

Example 4

Find the set of values of x for which:

a $2x - 5 < 7$

b $5x + 9 \geqslant x + 20$

c $12 - 3x < 27$

d $3(x - 5) > 5 - 2(x - 8)$

a $2x - 5 < 7$

$2x < 12$	Add 5 to both sides.
$x < 6$	Divide both sides by 2.

b $5x + 9 \geqslant x + 20$

$4x + 9 \geqslant 20$	Subtract x from both sides.
$4x \geqslant 11$	Subtract 9 from both sides.
$x \geqslant 2.75$	Divide both sides by 4.

For **c**, two approaches are shown:

c $12 - 3x < 27$

$-3x < 15$	Subtract 12 from both sides.
$x > -5$	Divide both sides by -3. (You therefore need to turn round the inequality sign.)

$12 - 3x < 27$

$12 < 27 + 3x$	Add $3x$ to both sides.
$-15 < 3x$	Subtract 27 from both sides.
$-5 < x$	Divide both sides by 3.
$x > -5$	Rewrite with x on LHS.

d $3(x - 5) > 5 - 2(x - 8)$

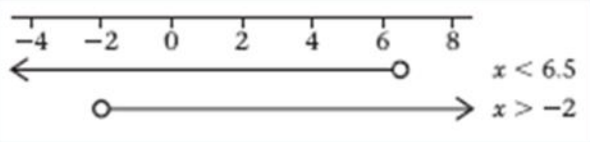
$3x - 15 > 5 - 2x + 16$	Multiply out (note: $-2 \times -8 = +16$).
$5x > 5 + 16 + 15$	Add 15 to both sides.
$5x > 36$	
$x > 7.2$	Divide both sides by 5.

Example 5

Find the set of values of x for which:

$$3x - 5 < x + 8 \text{ and } 5x > x - 8$$

$3x - 5 < x + 8$	$5x > x - 8$
$2x - 5 < 8$	$4x > -8$
$2x < 13$	$x > -2$
$x < 6.5$	



So the required set of values is $-2 < x < 6.5$.

Draw a number line to illustrate the two inequalities.

The 'hollow dots' at the end of each line show that the end value is <u>not</u> included in the set of values.

Show an included end value ($\leqslant$ or $\geqslant$) by using a 'solid dot' (●).

The two sets of values overlap (or intersect) where $-2 < x < 6.5$.

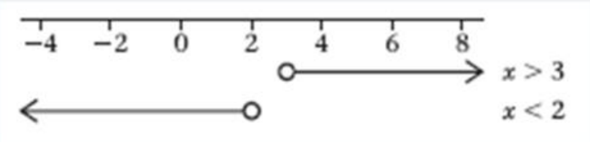
Notice here how this is written when x lies between two values.

Example 6

Find the set of values of x for which:

$$x - 5 > 1 - x \text{ and } 15 - 3x > 5 + 2x$$

$x - 5 > 1 - x$	$15 - 3x > 5 + 2x$
$2x - 5 > 1$	$10 - 3x > 2x$
$2x > 6$	$10 > 5x$
$x > 3$	$2 > x$
	$x < 2$



So there are no values of x for which both inequalities are true together.

Draw a number line. Note that there is no overlap between the two sets of values.

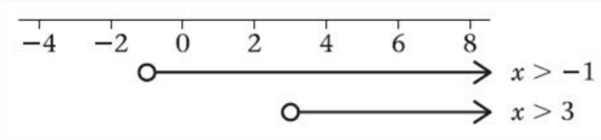
Example 7

Find the set of values of x for which:

$4x + 7 > 3$ and $17 < 11 + 2x$

$4x + 7 > 3$
$4x > -4$
$x > -1$

$17 < 11 + 2x$
$17 - 11 < 2x$
$6 < 2x$
$3 < x$
$x > 3$



Draw a number line. Note that the two sets of values overlap where $x > 3$.

So the required set of values is $x > 3$.

3.5 **To solve a quadratic inequality you**

- **solve the corresponding quadratic equation, then**
- **sketch the graph of the quadratic function, then**
- **use your sketch to find the required set of values.**

Example 8

Find the set of values of x for which $x^2 - 4x - 5 < 0$ and draw a sketch to show this.

$x^2 - 4x - 5 = 0$

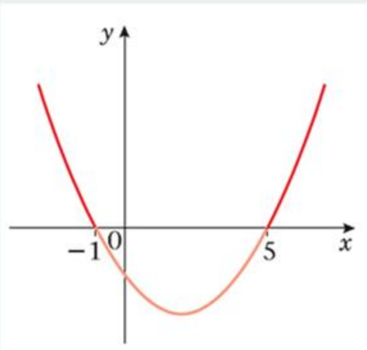
Quadratic equation.

$(x + 1)(x - 5) = 0$

Factorise (or use the quadratic formula). (See Section 2.5.)

$x = -1$ or $x = 5$

−1 and 5 are called critical values.



Your sketch does not need to be accurate. All you really need to know is that the graph is '∪-shaped' and crosses the x-axis at −1 and 5. (See Section 2.6.)

$x^2 - 4x - 5 < 0$ ($y < 0$) for the part of the graph below the x-axis, as shown by the paler part in the rough sketch.

So the required set of values is $-1 < x < 5$.

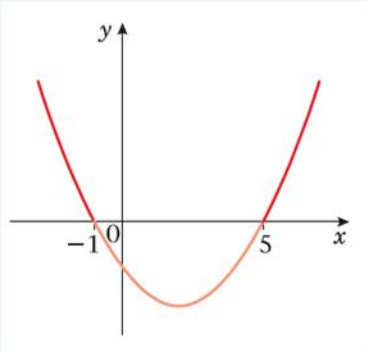
Example 9

Find the set of values of x for which $x^2 - 4x - 5 > 0$.

$x^2 - 4x - 5 = 0$

$(x + 1)(x - 5) = 0$

$x = -1$ or $x = 5$



The required set of values is $x < -1$ or $x > 5$.

The only difference between this example and the previous example is that it has to be greater than 0 (> 0). The solution would be exactly the same apart from the final stage.

$x^2 - 4x - 5 > 0$ ($y > 0$) for the part of the graph above the x-axis, as shown by the darker parts of the rough sketch in Example 8.

Be careful how you write down solutions like those on page 33.

$-1 < x < 5$ is fine, showing that x is between -1 and 5.

But it is wrong to write something like $5 < x < -1$ or $-1 > x > 5$ because x cannot be less than -1 and greater than 5 at the same time.

This type of solution (the darker parts of the graph) needs to be written in two separate parts, $x < -1, x > 5$.

Example 10

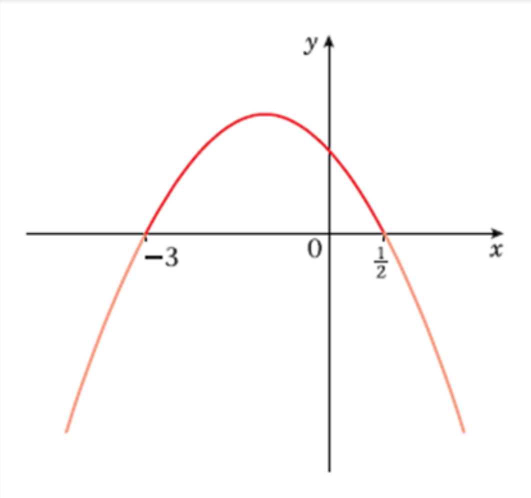
Find the set of values of x for which $3 - 5x - 2x^2 < 0$ and sketch the graph of $y = 3 - 5x - 2x^2$.

$3 - 5x - 2x^2 = 0$ — Quadratic equation.

$2x^2 + 5x - 3 = 0$ — Multiply by -1 (so it's easier to factorise).

$(2x - 1)(x + 3) = 0$

$x = \frac{1}{2}$ or $x = -3$ — $\frac{1}{2}$ and -3 are the critical values.



Since the coefficient of x^2 is negative, the graph is 'upside-down $\cup$-shaped' and crosses the x-axis at -3 and $\frac{1}{2}$ (see Section 2.6).

$3 - 5x - 2x^2 < 0$ ($y < 0$) for the outer parts of the graph, below the x-axis, as shown by the paler parts in the rough sketch.

So the required set of values is

$x < -3$ or $x > \frac{1}{2}$.

You may have to rearrange the quadratic inequality to get all the terms 'on one side' before you can solve it, as shown in the next example.

Example 11

Find the set of values of x for which $12 + 4x > x^2$.

Method 1: sketch graph

There are two possible approaches for Method 1, depending on which side of the inequality sign you put the expression.

$12 + 4x > x^2$

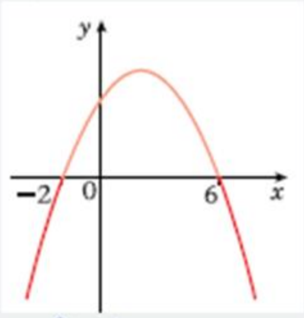
$12 + 4x - x^2 > 0$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x = -2$ or $x = 6$

Sketch of $y = 12 + 4x - x^2$



$12 + 4x - x^2 > 0$

Solution: $-2 < x < 6$

$12 + 4x > x^2$

$0 > x^2 - 4x - 12$

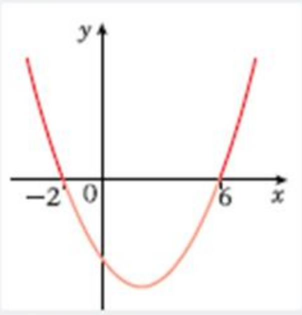
$x^2 - 4x - 12 < 0$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x = -2$ or $x = 6$

Sketch of $y = x^2 - 4x - 12$



$x^2 - 4x - 12 < 0$

Solution: $-2 < x < 6$

Find the set of values of x for which $12 + 4x > x^2$.

Method 2: table

$12 + 4x > x^2$

$0 > x^2 - 4x - 12$

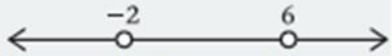
$x^2 - 4x - 12 < 0$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x = -2$ or $x = 6$

Use the critical values to split the real number line into sets.



	$x < -2$	$-2 < x < 6$	$x > 6$
$(x + 2)$	−	+	+
$(x - 6)$	−	−	+
$(x + 2)(x - 6)$	+	−	+

For each set, check whether the set of values makes the value of the bracket positive or negative.

For example, if $x < -2$, $(x + 2)$ is negative, $(x - 6)$ is negative, $(x + 2)(x - 6)$ is (neg) × (neg) = positive.

$x^2 - 4x - 12 < 0$

$(x + 2)(x - 6) < 0$

$(x + 2)(x - 6)$ is negative for $-2 < x < 6$

Solution: $-2 < x < 6$

TO DO:

1. Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

(Total 7 marks)

2.

(a) By eliminating y from the equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0.$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)
(Total 7 marks)

3.

(a) Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.

(2)

(b) Solve the simultaneous equations

$$x = 2y - 2,$$

$$x^2 = y^2 + 7.$$

(6)
(Total 8 marks)

4. Solve the simultaneous equations

$$y = x - 2,$$

$$y^2 + x^2 = 10.$$

(Total 7 marks)

---

5. Solve the simultaneous equations

$$x - 2y = 1,$$

$$x^2 + y^2 = 29.$$

(Total 6 marks)

---

6. Solve the simultaneous equations

$$x + y = 3,$$

$$x^2 + y = 15.$$

(Total 6 marks)

7.

In this question you must show detailed reasoning.

Andrea is comparing the prices charged by two different taxi firms.
Firm **A** charges £20 for a 5 mile journey and £30 for a 10 mile journey, and there is a linear relationship between the price and the length of the journey.
Firm **B** charges a pick-up fee of £3 and then £2.40 for each mile travelled.

Find the length of journey for which both firms would charge the same amount. **[4]**

8.

The specification for a rectangular car park states that the length x m is to be 5 m more than the breadth. The perimeter of the car park is to be greater than 32 m.

a Form a linear inequality in x.

The area of the car park is to be less than 104 m^2.

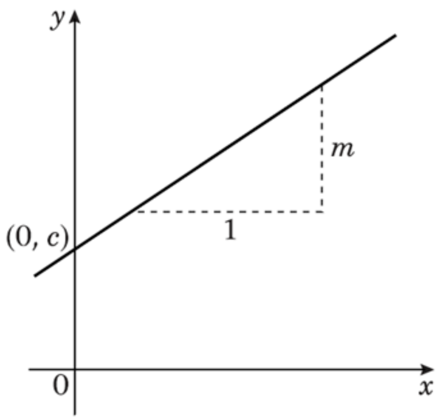
b Form a quadratic inequality in x.

c By solving your inequalities, determine the set of possible values of x.

(9)

5.1 You can write the equation of a straight line in the form $y = mx + c$ or $ax + by + c = 0$.

- **In the general form $y = mx + c$, m is the gradient and $(0, c)$ is the intercept on the y-axis.**



- **In the general form $ax + by + c = 0$, a, b and c are integers.**

Example 1

Write down the gradient and intercept on the y-axis of these lines:

a $y = -3x + 2$

b $4x - 2y + 5 = 0$

a $y = -3x + 2$

The gradient $= -3$ and the intercept on the y-axis $= (0, 2)$.

Compare $y = -3x + 2$ with $y = mx + c$.
From this, $m = -3$ and $c = 2$.

b $4x - 2y + 5 = 0$

Rearrange the equation into the form $y = mx + c$.

$4x + 5 = 2y$

Add $2y$ to each side.

So $2y = 4x + 5$

Put the term in y at the front of the equation.

$y = 2x + \frac{5}{2}$

Divide each term by 2, so that:

$2y \div 2 = y$
$4 \div 2 = 2$
$5 \div 2 = \frac{5}{2}$. (Do not write this as 2.5)

The gradient $= 2$ and the intercept on the y-axis $= (0, \frac{5}{2})$.

Compare $y = 2x + \frac{5}{2}$ to $y = mx + c$.
From this, $m = 2$ and $c = \frac{5}{2}$.

Example 2

Write these lines in the form $ax + by + c = 0$:

a $y = 4x + 3$ **b** $y = -\frac{1}{2}x + 5$

a $y = 4x + 3$

$0 = 4x + 3 - y$

So $4x - y + 3 = 0$

b $y = -\frac{1}{2}x + 5$

$\frac{1}{2}x + y = 5$

$\frac{1}{2}x + y - 5 = 0$

So $x + 2y - 10 = 0$

Rearrange the equation into the form $ax + by + c = 0$.

Subtract y from each side.

Collect all the terms on one side of the equation.

Add $\frac{1}{2}x$ to each side.

Subtract 5 from each side.

Multiply each term by 2 to clear the fraction.

Example 3

A line is parallel to the line $y = \frac{1}{2}x - 5$ and its intercept on the y-axis is (0, 1). Write down the equation of the line.

$y = \frac{1}{2}x + 1$

Remember that parallel lines have the same gradient.

Compare $y = \frac{1}{2}x - 5$ with $y = mx + c$, so $m = \frac{1}{2}$.

The gradient of the required line $= \frac{1}{2}$.

The intercept on the y-axis is (0, 1), so $c = 1$.

Example 4

A line is parallel to the line $6x + 3y - 2 = 0$ and it passes through the point (0, 3). Work out the equation of the line.

$6x + 3y - 2 = 0$

$3y - 2 = -6x$

$3y = -6x + 2$

$y = -2x + \frac{2}{3}$

The gradient of this line is -2.

The equation of the line is $y = -2x + 3$.

Rearrange the equation into the form $y = mx + c$ to find m.

Subtract $6x$ from each side.

Add 2 to each side.

Divide each term by 3, so that

$3y \div 3 = y$

$-6x \div 3 = -2x$

$2 \div 3 = \frac{2}{3}$. (Do not write this as a decimal.)

Compare $y = -2x + \frac{2}{3}$ with $y = mx + c$, so $m = -2$.

Parallel lines have the same gradient, so the gradient of the required line $= -2$.

(0, 3) is the intercept on the y-axis, so $c = 3$.

Example 5

The line $y = 4x - 8$ meets the x-axis at the point P. Work out the coordinates of P.

$y = 4x - 8$

Substituting,

$4x - 8 = 0$

$4x = 8$ — Add 8 to each side.

$x = 2$ — Divide each side by 4.

So $P(2, 0)$.

The line meets the x-axis when $y = 0$, so substitute $y = 0$ into $y = 4x - 8$.

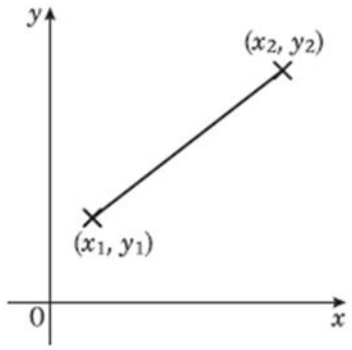
Rearrange the equation for x.

Add 8 to each side.

Divide each side by 4.

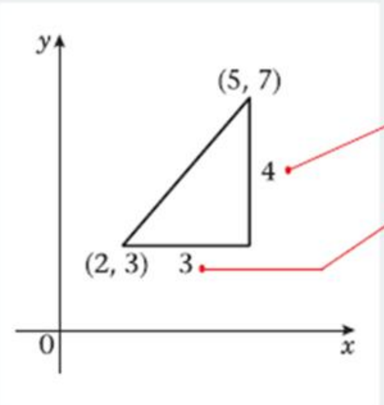
Always write down the coordinates of the point.

5.2 You can work out the gradient m of the line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) by using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.



Example 6

Work out the gradient of the line joining the points (2, 3) and (5, 7).



The gradient of the line is $\frac{4}{3}$.

Draw a sketch.

$7 - 3 = 4$

$5 - 2 = 3$

Remember the gradient of a line $= \dfrac{\text{difference in } y\text{-coordinates}}{\text{difference in } x\text{-coordinates}}$, so $m = \dfrac{7 - 3}{5 - 2}$.

This is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ with $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (5, 7)$.

Example 7

Work out the gradient of the line joining these pairs of points:

a $(-2, 7)$ and $(4, 5)$ **b** $(2d, -5d)$ and $(6d, 3d)$

a $m = \frac{5-7}{4-(-2)}$

Use $m = \frac{y_2 - y_1}{x_2 - x_1}$. Here $(x_1, y_1) = (-2, 7)$ and $(x_2, y_2) = (4, 5)$.

$= \frac{-2}{6}$

$-(-2) = +2$, so $4 + 2 = 6$

$= -\frac{1}{3}$

Remember to simplify the fraction when possible, so divide by 2.

$\frac{-1}{3}$ is the same as $-\frac{1}{3}$.

The gradient of the line is $-\frac{1}{3}$.

b $m = \frac{3d - (-5d)}{6d - 2d}$

Here $(x_1, y_1) = (2d, -5d)$ and $(x_2, y_2) = (6d, 3d)$.

$= \frac{8d}{4d}$

$-(-5d) = +5d$, so $3d + 5d = 8d$.

$= 2$

$8d \div 4d = 2$.

The gradient of the line is 2.

Example 8

The line joining $(2, -5)$ to $(4, a)$ has gradient -1. Work out the value of a.

$\frac{a-(-5)}{4-2} = -1$

Use $m = \frac{y_2 - y_1}{x_2 - x_1}$. Here $m = -1$, $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (4, a)$.

So $\frac{a+5}{2} = -1$

$a - (-5) = a + 5$

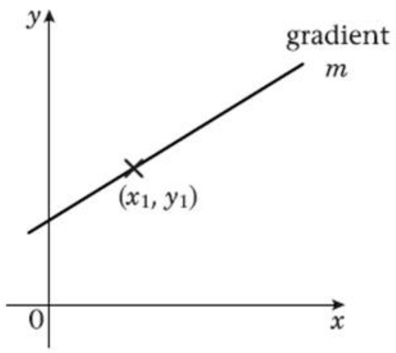
$a + 5 = -2$

Multiply each side of the equation by 2 to clear the fraction.

$a = -7$

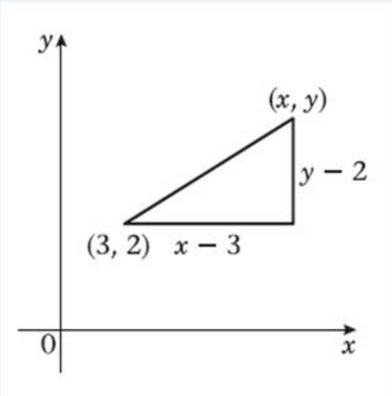
Subtract 5 from each side of the equation.

5.3 **You can find the equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) by using the formula $y - y_1 = m(x - x_1)$.**



Example 9

Find the equation of the line with gradient 5 that passes through the point (3, 2).



(x, y) is *any* point on the line.

The gradient = 5, so $\dfrac{y-2}{x-3} = 5$.

Multiply each side of the equation by $x - 3$ to clear the fraction, so that:

$$\frac{y-2}{x-3} \times \frac{x-3}{1} = y - 2$$

$$5 \times (x - 3) = 5(x - 3)$$

$y - 2 = 5(x - 3)$ — This is in the form $y - y_1 = m(x - x_1)$. Here $m = 5$ and $(x_1, y_1) = (3, 2)$.

$y - 2 = 5x - 15$ — Expand the brackets.

$y = 5x - 13$ — Add 2 to each side.

Example 10

Find the equation of the line with gradient $-\frac{1}{2}$ that passes through the point $(4, -6)$.

$y - (-6) = -\frac{1}{2}(x - 4)$ — Use $y - y_1 = m(x - x_1)$. Here $m = -\frac{1}{2}$ and $(x_1, y_1) = (4, -6)$.

So $y + 6 = -\frac{1}{2}(x - 4)$

$y + 6 = -\frac{1}{2}x + 2$ — Expand the brackets. Remember $-\frac{1}{2} \times -4 = +2$.

$y = -\frac{1}{2}x - 4$ — Subtract 6 from each side.

Example 11

The line $y = 3x - 9$ meets the x-axis at the point A. Find the equation of the line with gradient $\frac{2}{3}$ that passes through the point A. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

$y = 3x - 9$

$3x - 9 = 0$ — The line meets the x-axis when $y = 0$, so substitute $y = 0$ into $y = 3x - 9$.

$3x = 9$ — Rearrange the equation to find x.

$x = 3$

So $A(3, 0)$. — Always write down the coordinates of the point.

$y - 0 = \frac{2}{3}(x - 3)$ — Use $y - y_1 = m(x - x_1)$. Here $m = \frac{2}{3}$ and $(x_1, y_1) = (3, 0)$.

$y = \frac{2}{3}(x - 3)$ — Rearrange the equation into the form $ax + by + c = 0$.

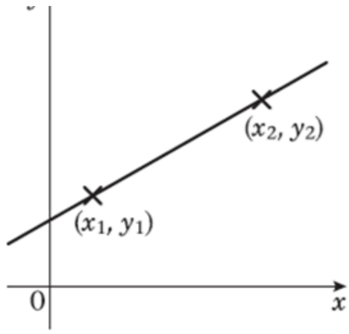
$3y = 2(x - 3)$ — Multiply by 3 to clear the fraction.

$3y = 2x - 6$ — Expand the brackets.

$-2x + 3y = -6$ — Subtract $2x$ from each side.

$-2x + 3y + 6 = 0$ — Add 6 to each side.

5.4 You can find the equation of the line that passes through the points with coordinates (x_1, y_1) and (x_2, y_2) by using the formula $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$.



Example 12

Work out the gradient of the line that passes through the points (5, 7) and (3, −1) and hence find the equation of the line.

$m = \frac{(-1) - 7}{3 - 5}$ — Use $m = \frac{y_2 - y_1}{x_2 - x_1}$. Here $(x_1, y_1) = (5, 7)$ and $(x_2, y_2) = (3, -1)$.

$= \frac{-8}{-2}$

So $m = 4$. — $-8 \div -2 = +4$

$y - 7 = 4(x - 5)$ — Use $y - y_1 = m(x - x_1)$. Here $m = 4$ and $(x_1, y_1) = (5, 7)$.

$y - 7 = 4x - 20$ — Expand the brackets.

$y = 4x - 13$ — Simplify into the form $y = mx + c$. Add 7 to each side.

Example 13

Use $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$ to find the equation of the line that passes through the points (5, 7) and (3, −1).

$$\frac{y - (-1)}{7 - (-1)} = \frac{x - 3}{5 - 3}$$

Use $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$.

Here $(x_1, y_1) = (3, -1)$ and $(x_2, y_2) = (5, 7)$.

(x_1, y_1) and (x_2, y_2) have been chosen to make the denominators positive.

So $$\frac{y + 1}{8} = \frac{x - 3}{2}$$

$$y + 1 = 4(x - 3)$$

Multiply each side by 8 to clear the fraction, so that:

$$8 \times \frac{y + 1}{8} = y + 1$$

$$8 \times \frac{x - 3}{2} = 4(x - 3)$$

$$y + 1 = 4x - 12$$

Expand the brackets.

$$y = 4x - 13$$

Subtract 1 from each side.

Example 14

The lines $y = 4x - 7$ and $2x + 3y - 21 = 0$ intersect at the point A. The point B has coordinates $(-2, 8)$. Find the equation of the line that passes through the points A and B. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

$y = 4x - 7,\ 2x + 3y - 21 = 0$

Solve the equations $y = 4x - 7$ and $2x + 3y - 21 = 0$ simultaneously to find the point A.

$2x + 3(4x - 7) - 21 = 0$ — Substitute $y = 4x - 7$ into $2x + 3y - 21 = 0$ to eliminate y.

$2x + 12x - 21 - 21 = 0$ — Expand the brackets.

$14x - 42 = 0$ — Collect like terms.

$14x = 42$ — Add 42 to each side.

$x = 3$ — Divide each term by 14.

Substituting,

$y = 4(3) - 7$ — Substitute $x = 3$ into either equation to find y. $y = 4x - 7$ is easier.

$y = 5$

So $A(3, 5)$. — Write down the coordinates of A.

$A(3, 5)$ and $B(-2, 8)$

$$\frac{y - 5}{8 - 5} = \frac{x - 3}{-2 - 3}$$

Use $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$. Here $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (-2, 8)$.

$$\frac{y - 5}{3} = \frac{x - 3}{-5}$$

Simplify the denominators.

$5(y - 5) = -3(x - 3)$ — Clear the fraction. Multiply each side by 15 so that

$$15 \times \frac{y - 5}{3} = 5(y - 5)$$

$$15 \times \frac{x - 3}{-5} = -3(x - 3)$$

$5y - 25 = -3x + 9$ — Expand the brackets. $-3 \times -3 = +9$

$3x + 5y - 25 = 9$ — Add $3x$ to each side.

$3x + 5y - 34 = 0$ — Subtract 9 from each side.

a $m = 3$

So the gradient of the perpendicular line is $-\frac{1}{3}$.

Use $-\frac{1}{m}$ with $m = 3$.

b $m = \frac{1}{2}$

So the gradient of the perpendicular line is

$$-\frac{1}{\left(\frac{1}{2}\right)}$$

$$= -\frac{2}{1}$$

$$= -2$$

Use $-\frac{1}{m}$ with $m = \frac{1}{2}$.

Remember $\frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a}$, so $\frac{1}{\left(\frac{1}{2}\right)} = \frac{2}{1}$.

c $m = -\frac{2}{5}$

So the gradient of the perpendicular line is

$$-\frac{1}{\left(-\frac{2}{5}\right)}$$

$$= -\left(-\frac{5}{2}\right)$$

$$= \frac{5}{2}$$

Use $-\frac{1}{m}$ with $m = -\frac{2}{5}$.

Here $\frac{1}{\left(\frac{2}{5}\right)} = \frac{5}{2}$, so $\frac{1}{\left(-\frac{2}{5}\right)} = -\frac{5}{2}$.

$-1 \times -\frac{5}{2} = +\frac{5}{2}$

Example 16

Show that the line $y = 3x + 4$ is perpendicular to the line $x + 3y - 3 = 0$.

$y = 3x + 4$

The gradient of this line is 3.

Compare $y = 3x + 4$ with $y = mx + c$, so $m = 3$.

$x + 3y - 3 = 0$

Rearrange the equation into the form $y = mx + c$ to find m.

$3y - 3 = -x$ — Subtract x from each side.

$3y = -x + 3$ — Add 3 to each side.

$y = -\frac{1}{3}x + 1$ — Divide each term by 3. $-x \div 3 = \frac{-x}{3} = -\frac{1}{3}x$.

The gradient of this line is $-\frac{1}{3}$. — Compare $y = -\frac{1}{3}x + 1$ with $y = mx + c$, so $m = -\frac{1}{3}$.

$3 \times -\frac{1}{3} = -1$ — Multiply the gradients of the lines.

The lines are perpendicular because the product of their gradients is -1.

Example 17

Work out whether these pairs of lines are parallel, perpendicular or neither:

a $y = -2x + 9$
$y = -2x - 3$

b $3x - y - 2 = 0$
$x + 3y - 6 = 0$

c $y = \frac{1}{2}x$
$2x - y + 4 = 0$

a $y = -2x + 9$

The gradient of this line is -2. — Compare $y = -2x + 9$ with $y = mx + c$, so $m = -2$.

$y = -2x - 3$ — Compare $y = -2x - 3$ with $y = mx + c$, so $m = -2$.

The gradient of this line is -2.

So the lines are parallel, since the gradients are equal. — Remember that parallel lines have the same gradient.

b $3x - y - 2 = 0$ — Rearrange the equation into the form $y = mx + c$.

$3x - 2 = y$ — Add y to each side.

So $y = 3x - 2$

The gradient of this line is 3. — Compare $y = 3x - 2$ with $y = mx + c$, so $m = 3$.

$x + 3y - 6 = 0$

$3y - 6 = -x$ — Subtract x from each side.

$3y = -x + 6$ — Add 6 to each side.

$y = -\frac{1}{3}x + 2$ — Divide each term by 3.

The gradient of this line is $-\frac{1}{3}$. — Compare $y = -\frac{1}{3}x + 2$ with $y = mx + c$, so $m = -\frac{1}{3}$.

So the lines are perpendicular as $3 \times \frac{1}{3} = -1$.

c $y = \frac{1}{2}x$ — Compare $y = \frac{1}{2}x$ with $y = mx + c$, so $m = \frac{1}{2}$.

The gradient of this line is $\frac{1}{2}$.

$2x - y + 4 = 0$ — Rearrange the equation into the form $y = mx + c$ to find m.

$2x + 4 = y$ — Add y to each side.

So $y = 2x + 4$

The gradient of this line is 2. — Compare $y = 2x + 4$ with $y = mx + c$, so $m = 2$.

The lines are not parallel as they have different gradients.

The lines are not perpendicular as $\frac{1}{2} \times 2 = 1$.

Example 18

Find an equation of the line that passes through the point $(3, -1)$ and is perpendicular to the line $y = 2x - 4$.

$y = 2x - 4$ — Compare $y = 2x - 4$ with $y = mx + c$.

$m = 2$

So the gradient of the perpendicular line is $-\frac{1}{2}$. — Use the rule $-\frac{1}{m}$ with $m = 2$.

$y - (-1) = -\frac{1}{2}(x - 3)$ — Use $y - y_1 = m(x - x_1)$. Here $m = -\frac{1}{2}$ and $(x_1, y_1) = (3, -1)$.

$y + 1 = -\frac{1}{2}x + \frac{3}{2}$ — Expand the brackets. $-\frac{1}{2} \times -3 = \frac{3}{2}$

$y = -\frac{1}{2}x + \frac{1}{2}$ — Subtract 1 from each side, so that $\frac{3}{2} - 1 = \frac{1}{2}$.

TO DO

1. The point $A(-6, 4)$ and the point $B(8, -3)$ lie on the line L.

(*a*) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(*b*) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer. **(3)**

---

2. The line l_1 passes through the point $(9, -4)$ and has gradient $\frac{1}{3}$.

(*a*) Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

The line l_2 passes through the origin O and has gradient -2. The lines l_1 and l_2 intersect at the point P.

(*b*) Calculate the coordinates of P. **(4)**

Given that l_1 crosses the y-axis at the point C,

(*c*) calculate the exact area of $\triangle OCP$. **(3)**

---

3. The line L has equation $y = 5 - 2x$.

(*a*) Show that the point P $(3, -1)$ lies on L. **(1)**

(*b*) Find an equation of the line perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

---

4. The line l_1 has equation $3x + 5y - 2 = 0$.

(*a*) Find the gradient of l_1. **(2)**

The line l_2 is perpendicular to l_1 and passes through the point $(3, 1)$.

(*b*) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants. **(3)**

---

5. (*a*) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants. **(4)**

The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

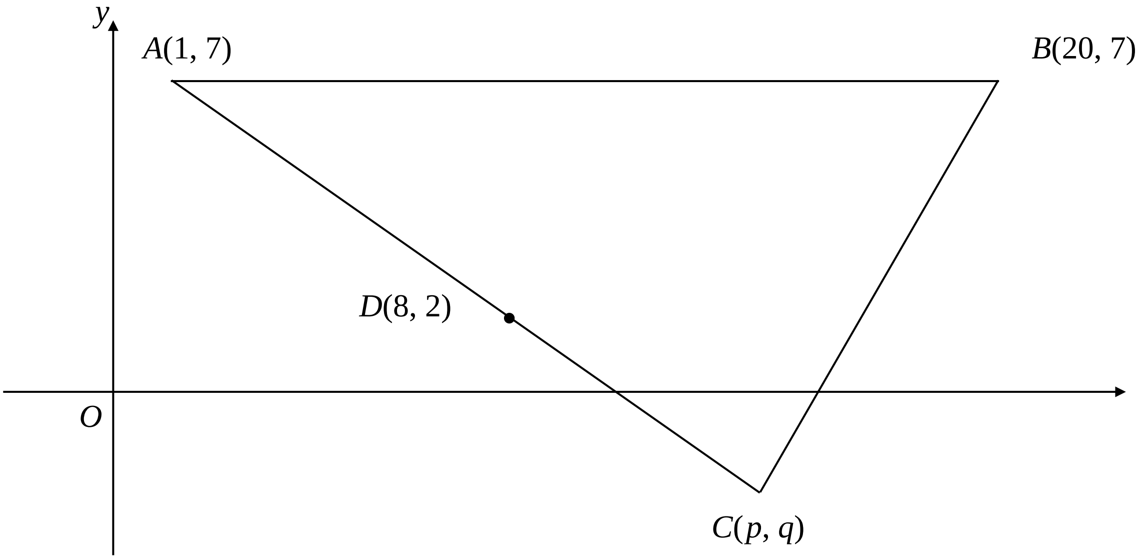
(*b*) Calculate the coordinates of S. **(5)**

(*c*) Show that the length of RS is $3\sqrt{5}$. **(2)**

(*d*) Hence, or otherwise, find the exact area of triangle PQR. **(4)**

---

6. **Figure 2**



The points $A(1, 7)$, $B(20, 7)$ and $C(p, q)$ form the vertices of a triangle ABC, as shown in Figure 2. The point $D(8, 2)$ is the mid-point of AC.

(*a*) Find the value of p and the value of q. **(2)**

The line l, which passes through D and is perpendicular to AC, intersects AB at E.

(*b*) Find an equation for l, in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

(*c*) Find the exact x-coordinate of E. **(2)**

---

7. The line l_1 has equation $y = 3x + 2$ and the line l_2 has equation $3x + 2y - 8 = 0$.

(*a*) Find the gradient of the line l_2. **(2)**

The point of intersection of l_1 and l_2 is P.

(*b*) Find the coordinates of P. **(3)**

The lines l_1 and l_2 cross the line $y = 1$ at the points A and B respectively.

(*c*) Find the area of triangle ABP. **(4)**

8.

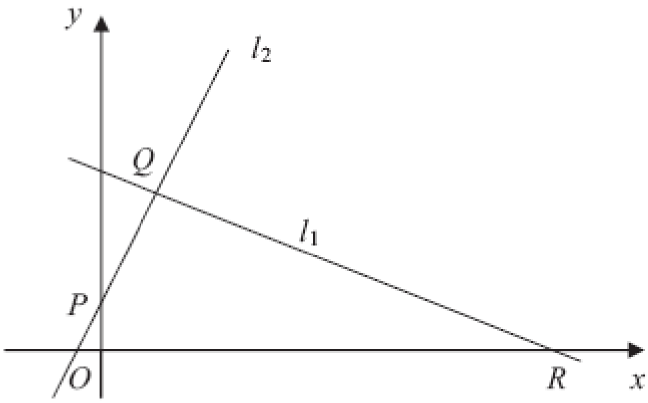


Figure 2

The points Q (1, 3) and R (7, 0) lie on the line l_1, as shown in Figure 2.

The length of QR is $a\sqrt{5}$.

(a) Find the value of a.

(3)

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2. Find

(b) an equation for l_2,

(5)

(c) the coordinates of P,

(1)

(d) the area of ΔPQR.

(4)

---

9. The line l_1 passes through the point $A(2, 5)$ and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$.

(3)

The point B has coordinates (–2, 7).

(b) Show that B lies on l_1.

(1)

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(d) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$

(4)

---

10. The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

---

11. (*a*) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

(*b*) Find the length of AB, leaving your answer in surd form.

(2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(*c*) Find the value of t.

(1)

(*d*) Find the area of triangle ABC.

(2)

---

12. The curve C has equation $y = x(5 - x)$ and the line L has equation $2y = 5x + 4$.

(*a*) Use algebra to show that C and L do not intersect.

(4)

(*b*) Sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes.

(4)

---

13. The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point $A(1, 4)$ lies on L_1, find

(*a*) the value of k,

(1)

(*b*) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1.

(*c*) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(*d*) Find the coordinates of B.

(2)

(*e*) Find the exact length of AB.

(2)

14.

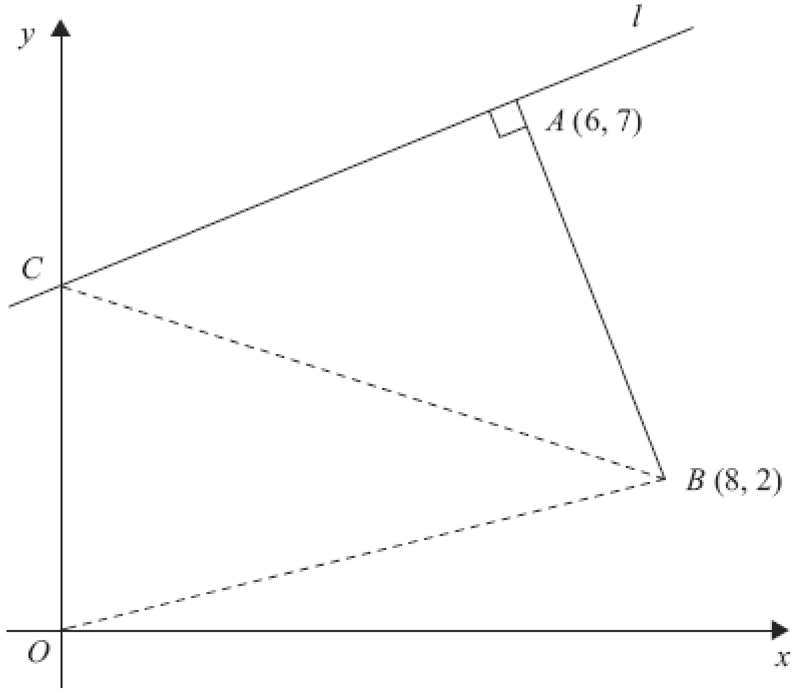


Figure 1

The points A and B have coordinates (6, 7) and (8, 2) respectively.

The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 1.

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C,

(2)

(c) the area of ΔOCB, where O is the origin.

(2)

---

15. The line l_1 passes through the points $A(-1, 4)$ and $B(5, -8)$

(a) Find the gradient of l_1

(2)

The line l_2 is perpendicular to the line l_1 and passes through the point $B(5, -8)$

(b) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

16. The line l_1 has equation $y = -2x + 3$.

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(*a*) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(*b*) Find the x-coordinate of A and the y-coordinate of B.

(2)

Given that O is the origin,

(*c*) find the area of the triangle OAB.

(2)

---

17. The curve C has equation

$$y = 9 - x^2$$

and the line l has equation

$$2y - 3x - 20 = 0$$

Use algebra to show that C and l do not intersect.

(4)

---

18. The line L_1 has equation $4y + 3 = 2x$.

The point A $(p, 4)$ lies on L_1.

(*a*) Find the value of the constant p. **(1)**

The line L_2 passes through the point C (2, 4) and is perpendicular to L_1.

(*b*) Find an equation for L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The line L_1 and the line L_2 intersect at the point D.

(*c*) Find the coordinates of the point D. **(3)**

(*d*) Show that the length of CD is $\frac{3}{2}\sqrt{5}$. **(3)**

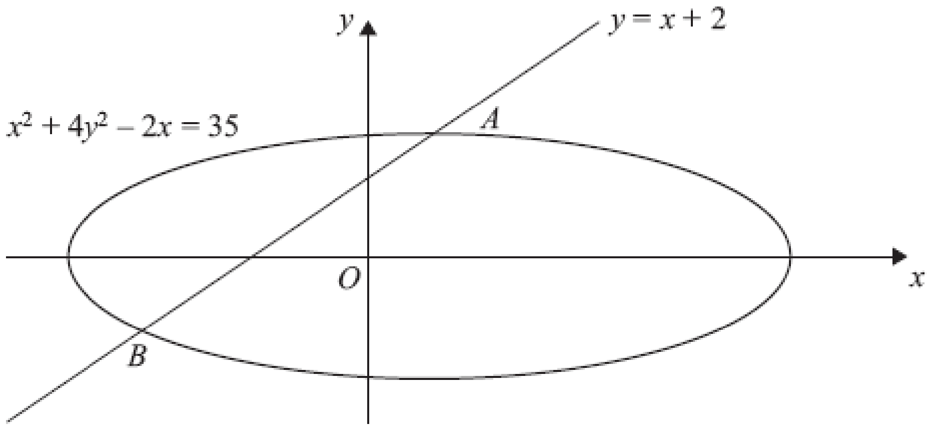
A point B lies on L_1 and the length of $AB = \sqrt{80}$.

The point E lies on L_2 such that the length of the line CDE = 3 times the length of CD.

(*e*) Find the area of the quadrilateral $ACBE$. **(3)**

EXTENSION:

1.



The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points A and B as shown in Figure 2.

(a) Find the coordinates of A and the coordinates of B.

(6)

(b) Find the distance AB in the form $r\sqrt{2}$, where r is a rational number.

(3)

2.

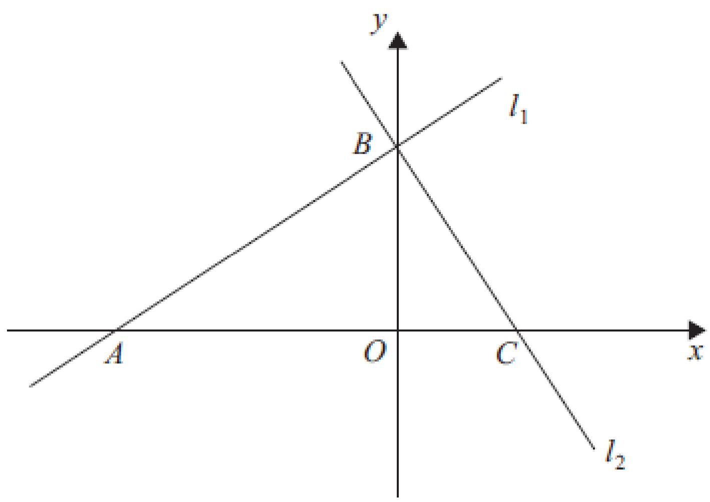


Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$.

(a) Find the gradient of l_1. **(1)**

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2. **(3)**

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC.

(4)

3.

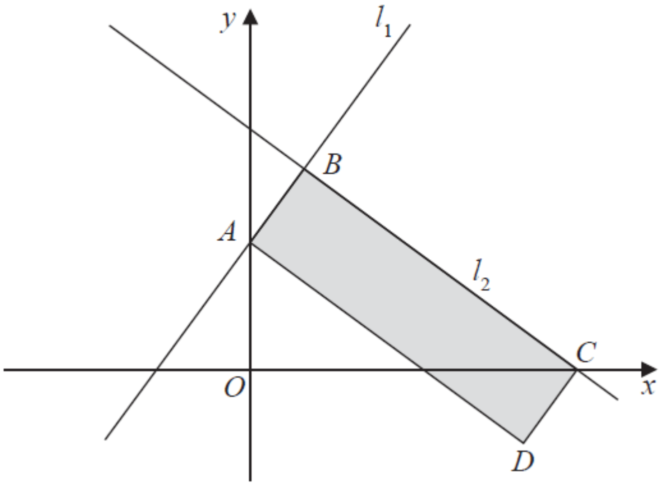


Figure 2

The straight line l_1 has equation $2y = 3x + 7$.

The line l_1 crosses the y-axis at the point A as shown in Figure 2.

(a) (i) State the gradient of l_1.

(ii) Write down the coordinates of the point A.

(2)

Another straight line l_2 intersects l_1 at the point B (1, 5) and crosses the x-axis at the point C, as shown in Figure 2.

Given that $\angle ABC = 90°$,

(b) find an equation of l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The rectangle $ABCD$, shown shaded in Figure 2, has vertices at the points A, B, C and D.

(c) Find the exact area of rectangle $ABCD$.

(5)

4.

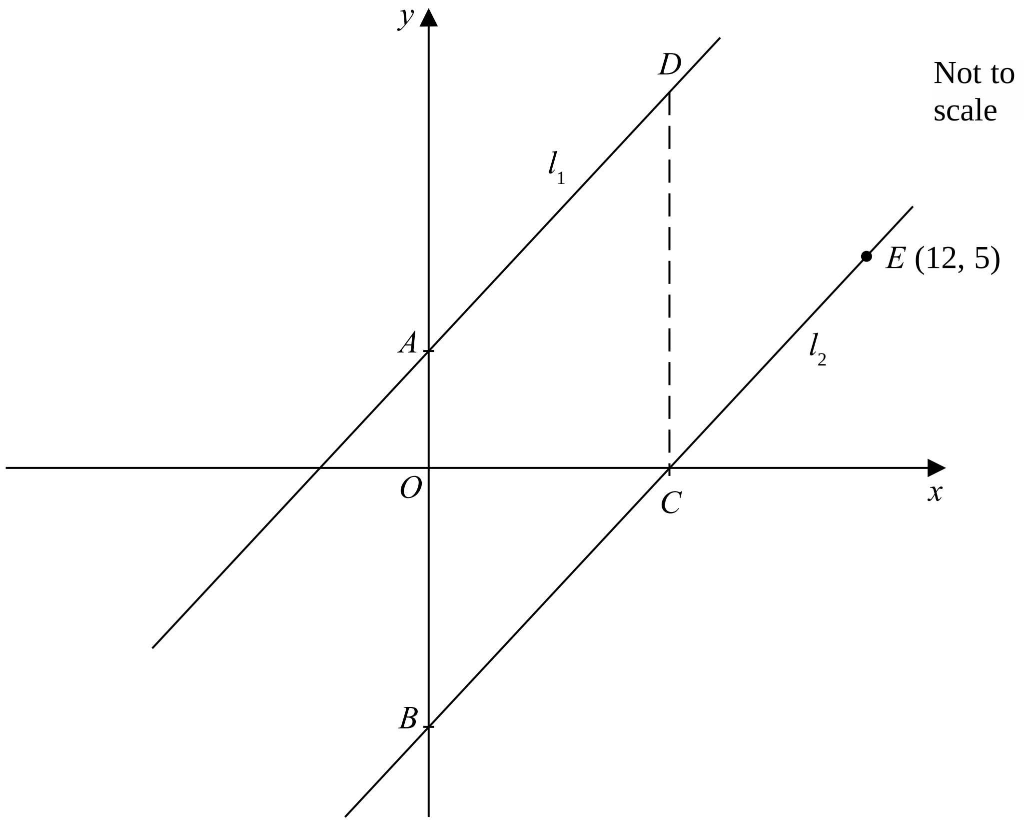


Figure 2

Figure 2 shows the straight line l_1 with equation $4y = 5x + 12$

(a) State the gradient of l_1

(1)

The line l_2 is parallel to l_1 and passes through the point E (12, 5), as shown in Figure 2.

(b) Find the equation of l_2. Write your answer in the form $y = m x + c$, where m and c are constants to be determined.

(3)

The line l_2 cuts the x-axis at the point C and the y-axis at the point B.

(c) Find the coordinates of

(i) the point B,

(ii) the point C.

(2)

The line l_1 cuts the y-axis at the point A.

The point D lies on l_1 such that $ABCD$ is a parallelogram, as shown in Figure 2.

(d) Find the area of $ABCD$. **(2)**